3D Geometry

Volume Word Problems (Solutions)

1. Joe has a cooler that is fifty centimetres long, twenty centimetres wide, and thirty centimetres high. He completely fills the cooler with his famous cookie dough ice cream to sell at a market. He sells the ice cream for zero point three six baht per cubic centimetre. How much money will Joe make if he sells all of his ice cream?

   The cooler is a right rectangular prism.

$$\begin{aligned} V_{cooler} &= lwh \\ &= 50\,\mathrm{cm} \times 20\,\mathrm{cm} \times 30\,\mathrm{cm} \\ &= 30{,}000\,\mathrm{cm}^3 \end{aligned}$$

$$\begin{aligned} profits &= price \times V_{cooler} \\ &= 0.36\,\text{฿}/\mathrm{cm}^3 \times 30{,}000\,\mathrm{cm}^3 \\ &= 10{,}800\,\text{฿} \end{aligned}$$

   $\therefore$ *Joe will make* 10,800 ฿ *if he sells all of his ice cream.*

2. A water tank is a rectangular prism that is three point two metres long, six point seven metres wide, and five point nine metres high. A solid metal box one point one metre long, three point two metres wide, and four point eight metres high is sitting inside the tank. The tank is filled with water. What is the volume of the water in the tank?

$$\begin{aligned} V_{tank} &= lwh \\ &= 3.2\,\mathrm{m} \times 6.7\,\mathrm{m} \times 5.9\,\mathrm{m} \\ &= 126.496\,\mathrm{m}^3 \end{aligned}$$

$$\begin{aligned} V_{box} &= lwh \\ &= 1.1\,\mathrm{m} \times 3.2\,\mathrm{m} \times 4.8\,\mathrm{m} \\ &= 16.896\,\mathrm{m} \end{aligned}$$

$$\begin{aligned} V_{water} &= V_{tank} - V_{box} \\ &= 126.496\,\mathrm{m}^3 - 16.896\,\mathrm{m} \\ &= 109.6\,\mathrm{m}^3 \end{aligned}$$

$\therefore$ *the volume of water in the tank is equal to* $109.6\,\mathrm{m}^3$.

3. I bought a box from the post office that has a volume of twenty four cubic centimetres. If the box is two centimetres long and six centimetres wide, how high is the box?

$$\begin{aligned} V_{box} &= lwh \\ h &= \frac{V_{box}}{lw} \\ &= \frac{24\,\mathrm{cm}^3}{2\,\mathrm{cm} \times 6\,\mathrm{cm}} \\ &= 2\,\mathrm{cm} \end{aligned}$$

$\therefore$ *the height of the box is* $2\,\mathrm{cm}$.

4. A rectangular-prism-shaped box is two metres by one point five metres by one point five metres. A shipping crate is packed with eighteen of these boxes. There is no extra space in the crate. What is the volume of the crate?

$$\begin{aligned} V_{box} &= lwh \\ &= 2\,\mathrm{m} \times 1.5\,\mathrm{m} \times 1.5\,\mathrm{m} \\ &= 4.5\,\mathrm{m}^3 \end{aligned}$$

$$\begin{aligned} V_{crate} &= 18 \times V_{box} \\ &= 18 \times 4.5\,\mathrm{m}^3 \\ &= 81\,\mathrm{m}^3 \end{aligned}$$

$\therefore$ *the volume of the crate is* $81\,\mathrm{m}^3$.

3D Geometry

Surface Area Word Problems (Solutions)

1. A birthday gift is fifty five centimetres long, forty centimetres wide, and five centimetres high. How much wrapping paper do you need to cover the whole gift?

$$\begin{aligned} A_{gift} &= 2A_B + hP \\ &= 2lw + 2h\,(l + w) \\ &= 2 \times 5\,\mathrm{cm} \times 40\,\mathrm{cm} + 2 \times 5\,\mathrm{cm} \times (5\,\mathrm{cm} + 40\,\mathrm{cm}) \\ &= 850\,\mathrm{cm}^3 \end{aligned}$$

$\therefore$ *You need* $850\,\mathrm{cm}^3$ *of wrapping paper to cover the whole gift.*

2. Jay wants to paint the walls of a room. The room is six metres long, eight metres wide, and three metres high. There is a door that is one metre wide and two metres tall. One litre of paint can paint four point five square metres of space and costs two hundred and twenty five bahts. How much will it cost to paint the room twice?

$$\begin{aligned} A_{room} &= 2A_B + hP \\ &= 2lw + 2h\,(l + w) \\ &= 2 \times 6\,\mathrm{m} \times 8\,\mathrm{m} + 2 \times 3\,\mathrm{m} \times (6\,\mathrm{m} + 8\,\mathrm{m}) \\ &= 180\,\mathrm{m}^2 \end{aligned}$$

$$\begin{aligned} A_{ceiling\ and\ floor} &= 2lw \\ &= 2 \times 6\,\mathrm{m} \times 8\,\mathrm{m} \\ &= 96\,\mathrm{m}^2 \end{aligned}$$

$$\begin{aligned} A_{door} &= wh \\ &= 1\,\mathrm{m} \times 2\,\mathrm{m} \\ &= 2\,\mathrm{m}^2 \end{aligned}$$

$$\begin{aligned} A_{walls} &= A_{room} - A_{ceiling\ and\ floor} - A_{door} \\ &= 180\,\mathrm{m}^2 - 96\,\mathrm{m}^2 - 2\,\mathrm{m}^2 \\ &= 82\,\mathrm{m}^2 \end{aligned}$$

$$\begin{aligned} paint &= 2 \times \frac{1\,\mathrm{L}}{4.5\,\mathrm{m}^2} \times A_{walls} \\ &= 2 \times \frac{1\,\mathrm{L}}{4.5\,\mathrm{m}^2} \times 82\,\mathrm{m}^2 \\ &\approx 36.5\,\mathrm{L} \end{aligned}$$

$$\begin{aligned} cost &= price \times paint \\ &\approx 225\,\text{฿}/\mathrm{L} \times 36.5\,\mathrm{L} \\ &= 8212.50\,\text{฿} \end{aligned}$$

$\therefore$ *it will cost* 8212.50 ฿ *to paint the room twice.*

3. The radius of the Earth is six thousand three hundred and seventy one kilometres. What is the surface area of the Earth if the Earth is a perfect sphere?

$$\begin{aligned} A_{Earth} &= 4\pi r^2 \\ &= 4 \times \pi \times (6371\,\text{km})^2 \\ &\approx 510{,}064{,}471.9\,\text{km}^2 \end{aligned}$$

$\therefore$ *the surface area of the Earth, if the Earth is a perfect sphere, is approximately* $510{,}064{,}471.9\,\text{km}^2$.

4. A cylindrical bottle has a radius of five centimetres and a height of thirty two centimetres. What is the surface area of the bottle?

$$\begin{aligned} A_{bottle} &= 2\pi r^2 + 2\pi rh \\ &= 2 \times \pi \times (5\,\mathrm{cm})^2 + 2 \times \pi \times 5\,\mathrm{cm} \times 32\,\mathrm{cm} \\ &\approx 477.1\,\mathrm{cm}^2 \end{aligned}$$

$\therefore$ *the surface area of the bottle is approximately* $477.1\,\mathrm{cm}^2$.

3D Geometry

Volume and Surface Area Word Problems (Solutions)

1. A cube has a volume of thirty two cubic kilometres. What is the surface area of the cube?

$$\begin{aligned} V_{cube} &= s^3 \\ s &= \sqrt[3]{V_{cube}} \\ &= \sqrt[3]{32\,\mathrm{km}^3} \\ &\approx 3.2\,\mathrm{km} \end{aligned}$$

$$\begin{aligned} A_{cube} &= 6s^2 \\ &\approx 6\,(3.2\,\mathrm{km})^2 \\ &= 61.44\,\mathrm{km}^2 \end{aligned}$$

$\therefore$ *the surface area of the cube is approximately* $61.44\,\mathrm{km}^2$.

2. Three tennis balls with a diametre of eight centimetres are in a cylindrical container. What is the surface area of the container?

$$\begin{aligned}
h_{container} &= 3h_{tennis\ ball} \\
&= 3 \times 8\,\text{cm} \\
&= 24\,\text{cm}
\end{aligned}$$

$$\begin{aligned}
r_{container} &= \frac{1}{2}d_{tennis\ ball} \\
&= \frac{1}{2} \times 8\,\text{cm} \\
&= 4\,\text{cm}
\end{aligned}$$

$$\begin{aligned}
A_{container} &= 2\pi r^2 + 2\pi rh \\
&= 2 \times \pi \times (4\,\text{cm})^2 + 2 \times \pi \times 4\,\text{cm} \times 24\,\text{cm} \\
&\approx 703.7\,\text{cm}^2
\end{aligned}$$

$\therefore$ *the surface area of the container is approximately* $703.7\,\text{cm}^2$.

3. A sphere fits perfectly inside of a cube. The cube has a volume of twelve cubic centimetres. What is the surface area of the sphere?

$$\begin{aligned} V_{cube} &= s^3 \\ s &= \sqrt[3]{V_{cube}} \\ &= \sqrt[3]{12\,\mathrm{cm}^3} \\ &\approx 2.9\,\mathrm{cm} \end{aligned}$$

$$\begin{aligned} r_{sphere} &= \frac{1}{2}s \\ &\approx \frac{1}{2} \times 2.9\,\mathrm{cm} \\ &= 1.45\,\mathrm{cm} \end{aligned}$$

$$\begin{aligned} A &= 4\pi r^2 \\ &\approx 4 \times \pi \times (1.45\,\mathrm{cm})^2 \\ &= 26.4\,\mathrm{cm}^3 \end{aligned}$$

$\therefore$ *the surface area of the sphere is approximately* $26.4\,\mathrm{cm}^3$.

4. A spherical container full of water has a surface area of fifty square metres. Half of the water is poured into and complete fills a cubical container. What is the surface area of the cubical container?

$$
\begin{aligned}
A_{sphere} &= 4\pi r^2 \\
r &= \sqrt{\frac{A_{sphere}}{4\pi}} \\
&= \sqrt{\frac{50\,\mathrm{m}^2}{4 \times \pi}} \\
&\approx 2.0\,\mathrm{m}
\end{aligned}
$$

$$
\begin{aligned}
V_{sphere} &= \frac{4}{3}\pi r^3 \\
&\approx \frac{4}{3} \times \pi \times (2.0\,\mathrm{m})^3 \\
&\approx 33.5\,\mathrm{m}^3
\end{aligned}
$$

$$
\begin{aligned}
V_{cube} &= \frac{1}{2} V_{sphere} \\
&= \frac{1}{2} \times 33.5\,\mathrm{m}^3 \\
&= 16.75\,\mathrm{m}^3
\end{aligned}
$$

$$
\begin{aligned}
V_{cube} &= c^3 \\
c &= \sqrt[3]{V_{cube}} \\
&\approx \sqrt[3]{16.75\,\mathrm{m}^3} \\
&\approx 2.6\,\mathrm{m}
\end{aligned}
$$

$$
\begin{aligned}
A_{cube} &= 6c^2 \\
&\approx 6 \times (2.6\,\mathrm{m})^2 \\
&= 40.56\,\mathrm{m}^2
\end{aligned}
$$

$\therefore$ *the surface area of the cubical container is approximately* $40.56\,\mathrm{m}^2$.